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Pricing Average Price Options for the 1990 Mexican and Venezuelan Recapture Clauses

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and
Sweder van Wijnbergen

Pricing models are developed to value the recapture clauses in the 1990 Mexican and Venezuelan debt restructuring agreements. The current values of the recapture clauses are less than one-quarter of the maximum contractually possible and decrease as the standard deviation of the oil price increases.

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Policy, Research, and External Affairs

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This paper — a joint product of the Debt and International Finance Division, International Economics Department and the Country Operations 1 Division, Country Department II, Latin America and the Caribbean Regional Office — is part of a larger effort in PRE to study the benefits and costs of contingent external debt contracts and debt and debt service reduction operations. Copies are available free from the World Bank, 1818 H Street NW, Washington, DC 20433. Please contact Sheilah King-Watson, room S8-025, extension 31047 (14 pages).

Making restructured sovereign debt obligations contingent on exogenous factors (such as world oil prices) allows some of the risk to be transferred to creditors who have comparative advantage in carrying the risks — as they can diversify them in capital markets.

Contingencies also increase the borrowers' likelihood of fulfilling their (new or restructured) external obligations — and can improve the heavily indebted countries' incentives to invest and adjust, increasing further the likelihood they will service their external obligations.

The 1986 agreement between Mexico and commercial banks included some contingency facilities where new money would be forthcoming if international oil prices fell below a certain level or when Mexico's growth rate was to fall short of a certain rate. In the 1990 Mexico and Venezuela agreements, future debt service obligations were indexed to factors largely exogenous to the countries — the so-called recapture clauses.

Under the recapture clause in Mexico, 30 percent of the extra oil revenues Mexico gets if the price of oil rises above $14 per barrel (adjusted for U.S. inflation) will accrue to the banks that have granted debt service relief. (This amount is not to exceed 3 percent of the nominal value of the debt exchanged for these bonds, in any year.) The value of the recapture clause at maturity depends on three variables: how much oil Mexico exports, how oil prices behave, and the behavior of inflation rates.

Export volume is not a factor in Venezuela's 1990 recapture clause, which in other ways is similar to Mexico's.

Claessens and van Wijnbergen develop pricing models for options written on average prices and contingent contracts used in sovereign debt restructuring. They use the models to price the recapture clauses in the 1990 Mexican and Venezuelan debt restructuring agreements.

The current values of the recapture clauses are less than one-quarter of the maximum contractually possible and decrease as the standard deviation of the oil price increases.

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Pricing Average Price Options for the 1990 Mexican and Venezuelan Recapture Clauses

by
Stijn Claessens
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* Copies of the computer programs used are available on request from the authors.

I. Introduction

It has long been recognized that debt obligations and provisions of new financing which are contingent with respect to a set of exogenous factors can play a useful role in international finance for developing countries. Making debt obligations contingent on some exogenous factors can allow for not only a transfers of risk to the creditors which have comparative advantage in carrying these risks (as they can diversify these risks in the international capital markets), but also for a greater likelihood of a borrower fulfilling his (new or restructured) external obligations. For a heavily indebted country, contingencies can further improve the country's incentives to invest and adjust, leading to a higher likelihood of servicing its external obligations.

The 1986 agreement between Mexico and its commercial banks included some contingency facilities where new money would be forthcoming in case international oil prices fell below a certain level or when Mexico's growth rate was to fall short of a certain rate. Contingencies in the context of debt reduction agreements, where future reduced debt obligations are indexed with respect to factors (largely) exogenous to the country, were for the first time used in the 1990 Mexico agreement (the so-called recapture clause). The 1990 Venezuela agreement incorporates a similar recapture clause.[1] Other contingencies have been introduced in international finance through commodity bonds, which have been used on a larger scale in recent years.

The pricing of contingent contracts has been widely studied in the field of finance, where the payoff structure of so-called derivative assets often depend on the behavior of an underlying state variable. The methodology developed in finance has however not seen an empirical application to the pricing of contingencies (recapture clauses) in international debt contracts. To the contrary, many times, methods useful in certainty frameworks have been applied to value contracts in international finance with contingency features. This note bridges the gap and presents a simple methodology to price contingencies in international finance with a particular application to the recapture clauses in the 1990 Mexico and 1990 Venezuela agreements.

II. Pricing Debt Claims and Recapture Clauses

We first develop a general model of pricing of a commercial bank debt claim on a country using option pricing.[2]/ The setup is the following. Due to uncertainty in the country's export earnings, import requirements and net capital in-or outflows, the net amount of financing available each period to service

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[1]These recapture clauses were written, however, for up-side contingencies. Other clauses (for new financing) were intended to be included in the 1990 Mexico agreement for downside contingencies, but never materialized.

[2]For a further description of the option pricing methodology see Claessens and van Wijnbergen (1989).

foreign commercial debt is uncertain.[3]/ Each period the country pays as much as its financial resources allow to the commercial banks, but never more than its contractual obligations in the period. Consequently, repayments may fall short of commercial debt service obligations due.

We can represent this repayment behavior by the following:

(1a) $R^*(t) = \min\ [R_t, FX_t]$

with $R^*(t)$ equal to the repayment in period t; R_t the contractual debt service in period t; and FX_t the resources available to service commercially held debt, all in period t. (1a) can be rearranged to yield:

(1b) $R^*(t) = R_t - \max[0, R_t - FX_t]$

But $\max[0, R_t - FX_t]$ equals the payoff at maturity of a (European) put, with a strike price of R_t, which is written on the value of the foreign exchange available, FX_t. [4]/ Thus equation 1b shows that the uncertain repayment can be represented by a certain repayment R_t minus a put, with a strike price of R_t, which is written on the value of the foreign exchange available, FX_t.

Now that we have replicated the payoff stream, it is easy to calculate the current value of the uncertain payoff stream as the current value of the certain future obligation R_t minus the current value of the put. This current value of the uncertain payoff stream equals thus the discounted value of R_t, $\exp(-rt)*R_t$ (where r is the (continuously compounding) interest rate), minus the current

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[3]The uncertainty in the amount of resources used to service foreign obligations can be due to ability to pay as well as willingness to pay factors. For convenience, we lump these factors together and assume that the creditors have appropriability of any resources falling short of contractual debt service, or, alternatively and equivalently, that the country is a perfectly willing, but sometimes unable payer.

[4] The state variable FX is a non-traded asset and not as such priced in the market. But if the state variable is spanned by other traded instruments, one can price the non-traded asset and all results go through identically as in the case of traded assets. See also section III. By defining the repayment behavior in period t as in equation 1a, we have implicitly made the assumption that claims which are not met in previous periods will not be added on (rescheduled) to the stock of claims already outstanding. This contrasts with Kharas et all. (1987), Cohen (1989) and Dixit and Bartolini (1990) who assume that all unpaid claims are always rescheduled. The reality is between theirs and our extremes: not all unpaid debt will be rescheduled at original terms and some may be forgiven. The assumption we made that unmet claims are not carried over was with the numerical application in mind.

value P of a put with an exercise price of R_t, written on FX_t. [5] If $V(R_t)$ is the present value of the claim, we can represent this as:

$$(2) \quad V(R_t) = \exp(-rt)*R_t - P(FX_t,R_t,r,t,\sigma).$$

where $P(FX_t,R_t,r,t,\sigma)$ is the current value of a put written on FX_t with exercise price R_t, interest rate r, maturity t and standard deviation of FX of σ. If one furthermore assumes that FX behaves lognormally, the pricing of the put can be done using the Black and Scholes option pricing formula (see Black and Scholes (1973)). [6] P is then equal to the following expression:

$$(3) \quad P(FX_t,R_t,r,t,\sigma) = - FX_0*\exp((\mu-r)t)*N(d1) + \exp(-rt)R*N(d2)$$

where

$$d1 = [-\log(FX_0*\exp(\mu t)/R_t) - (\sigma^2/2)*t]/(\sigma\sqrt{t})$$

$$d2 = d1 + \sigma\sqrt{t}$$

μ = the drift in FX_t over the period 0..t [7]

The option pricing methodology can also be used to price recapture clauses, where the future payments obligations of the country are in some fashion indexed to the amount of foreign exchange available in a particular period. Take the following recapture clause (which is similar to the one of the 1990 Mexico

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[5] The formula assumes a constant interest rate r for notational convenience only. The empirical application presented below allows for different maturity structures of interest rates.

[6] Other density functions can easily be incorporated using numerical integration techniques.

[7] The formula assumes a constant drift μ for notational convenience only. The empirical application presented below allows for time varying drift parameter μ. The valuation formula differs from the Black-Scholes equation in that we do not assume μ=r. The valuation formula we use is therefore closer to the pricing of a commodity option written on the futures prices of the underlying commodity. Black (1976, page 177) shows that the value of such a commodity option written on the futures price can be obtained from the original Black and Scholes formula by substituting xe^{-rt} for x everywhere in the original formula, where x stands for the futures price. In the application here we use the time-varying drift parameter μ to incorporate the deviation of the futures price at each maturity date from the current spot price. In this way, we model a forward curve of the net amount of foreign financing available to service commercial bank debt, and thus a forward (or futures) oil pricing curve. Historically, the forward yield curve for oil has been upward sloping with a slope less than the nominal interest rate. This is due to, among others, the relative levels of convenience yields and carrying costs. In the application here, this would imply that μ is less than r for each period. Forward prices are now avalable for approximately 10-15 years. The volatility of futures (or forward) oil prices is of the same order of magnitude as the volatility of spot oil prices.

agreement). Assume, that the clause stipulates that the creditors are entitled (in exchange for a certain amount of debt reduction at time zero or for an amount of cash), whenever foreign exchange exceeds a certain level L, to a share α of the excess foreign exchange over L in every period after time τ up to time T. The maximum amount that creditors can receive per period under this sharing rule is limited by an amount M. [8]/

Such a sharing rule can easily be represented in terms of option terminology: the creditors hold a fraction α of a series of calls that are written on FX with exercise prices L, maturity dates $\tau+1$, $\tau+2,..,T$, and are short a fraction α of a series of calls that are written on FX with exercise price $U=L+M/\alpha$ and maturity dates $\tau+1$, $\tau+2,..,T$.

To see the equivalence between the sharing rule and the portfolio of options just described, consider the payoff structure for the recapture clause, which we call I.

$$(4) \qquad I = \Sigma_{\tau'>\tau} \max[\alpha\max[FX_{\{\tau'\}}-L,0],M]$$

$$= \Sigma_{\tau'>\tau}\ \alpha*(\max[FX_{\{\tau'\}}-L,0] - \max[FX_{\{\tau'\}}-U,0]);\ U=L+M/\alpha$$

The expressions in the two brackets in the last equation are the two calls mentioned above, with exercise prices L and $U=L+M/\alpha$. It can be verified that the two calls yield the desired payoff function, $I_t = \max[\alpha\max[FX_{\{\tau'\}}-L,0],M]$. Holding long a fraction α of the first call pays $\alpha\max[FX_t-L,0]$: whenever FX_t falls below R_t, the call is out of the money and its value is equal to 0; and whenever FX_t is above L, the call is in the money and thus pays $\alpha(FX_t - L)$. Being short a fraction α of the second call pays $-\alpha\max[FX_t - U,0]$. The difference between the payoff of the two calls is thus the payoff of the recapture clause. The value of the two individual calls can easily be calculated using option pricing techniques, in particular the Black-Scholes formula.

We can also represent this equivalency graphically. Figure 1 shows on the horizontal axis the value of the foreign exchange available at maturity date and on the vertical axis payoffs for the two calls and the recapture clause. The figure has three lines which have a slope of one, that represent the payoff functions of respectively holding long a call with exercise price L ("long"), holding long a call with exercise price U, and, its mirror image, being short a call with exercise price U ("short). In addition, a line with slope α, intersected by two horizontal lines at values 0 and M, is drawn. The difference between the fraction α of the payoff of the long call and the fraction α of the payoff of the short call is thus represented by the segment of the line with slope α between the points (L,0) and (U,M) and is equal to the payoff of the recapture clause for a given FX at the maturity date. As will be clear, the value of the recapture clause--the (probability weighted) shaded area--will depend on the probability density of FX over the segment between L and U.

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[8] L, M and α can be made time dependent. In addition, L, M and α can be made dependent on other stochastic variables, such as world inflation rates in case of indexed clauses (see the application below).

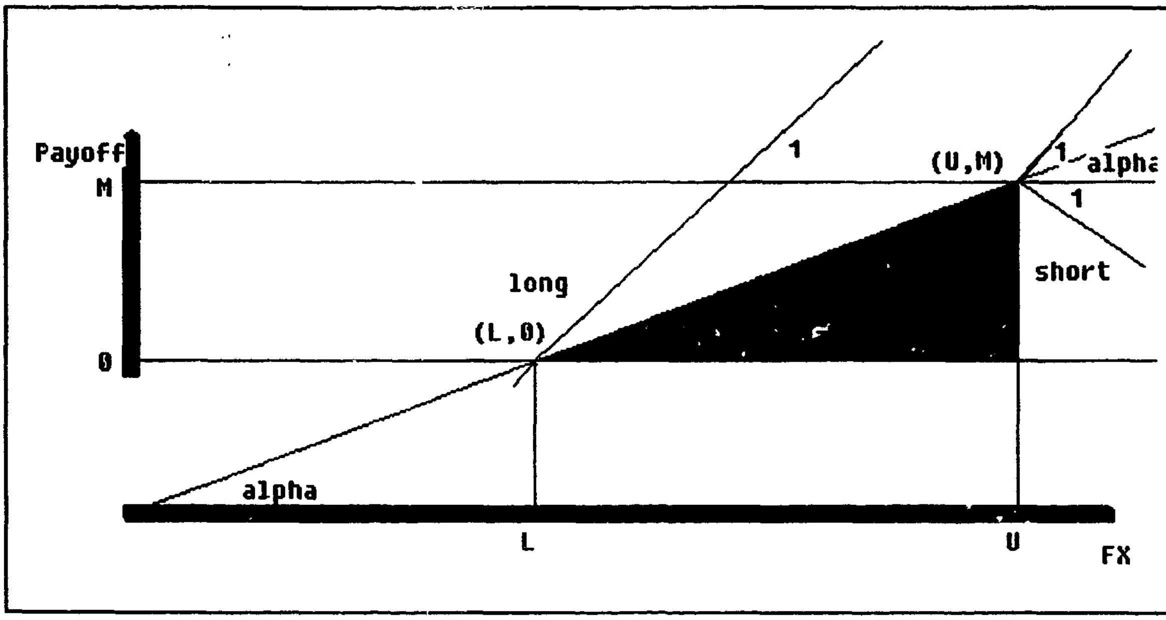

Figure 1

In general, recapture clauses which depend in a more complicated manner on FX can also be handled using option pricing methodology. One complication which is common to both the Venezuela and Mexico recapture clause is that the underlying state variable is not the spot (oil) price at the maturity date, an asset price similar to a stock price used for standard option pricing, but instead a flow measure, involving an average (of a price) over a certain time period.[9]/ For this reason, we will develop here a general pricing formula for options which are written on the average of a state variable and use this in the applications for Mexico and Venezuela.

Consider an European-style call option (i.e., an option which is only exercisable at the maturity date) with maturity date T and with exercise price X, which is written on the average of the price S_t over the time interval [T-1,T], where the average price is calculated using very high frequency (infinite number of) observations. At time T-1, the spot price S_{T-1} will of course be known with certainty, and the value of the option will depend only on the expected average price over the period [T-1,T] (and of course other inputs for the option price such as the interest rate and the exercise price). If the parameters of the stochastic process of S over period [T-1,T] are known, then it will be clear that the expectation of the average price over [T-1,T], call it A_{T-1}, given S_{T-1} will

[9]We will see in the two applications that the average involved either the (average) foreign exchange earnings over the preceding 12 months--in case of Mexico--or the average price received over the preceding six months--in case of Venezuela.

not be stochastic.[10]/ At time T–1, there will be no instantaneous uncertainty about A_{T-1}, and the value of the option will therefore be known with certainty. As time goes on, more will be known about the average price as observations on S will come in— which will of course be reflected in the price of the option, but instantaneously the drift in A and the option price is known.

In case the price process S has a lognormal distribution with drift parameter μ, the expected average price over period [T–1,T] will just be:

(4) $$A_{T-1} = S_{T-1}*[\exp(\mu(T-(T-1)))-1]/\mu = S_{T-1}*[\exp(\mu)-1]/\mu.$$ [11]/

The value of a call option written on the average price with an exercise price X at time T–1, call it $Z(A_{T-1},T-1)$, will then be:

(5) $$Z(A_{T-1},T-1) = \exp(-r(T-(T-1))*\max[S_{T-1}*[\exp(\mu)-1]/\mu-X,0].$$

Notice that this equation does not involve taken any expected values. Equation (5) can be rewritten, by bringing a factor outside the max-operator, as:

(5′) $$Z(A_{T-1},T-1) = \max[S_{T-1}-\mu/[\exp(\mu)-1]*X,0]*[\exp(\mu)-1]/\mu*\exp(-r)$$

Equation (5′) shows that the value at time T–1 of an European call option written on the average price over period [T–1,T], is equal to the payoff at maturity of a European call option written on S_{T-1} with exercise price $\mu/[\exp(\mu)-1]*X$ and maturity date T–1, multiplied by the factor $\exp(-r)*[\exp(\mu)-1]/\mu$. At any time t before T–1, we can use normal option pricing techniques to price this last option by just specifying the correct exercise price and maturity date. Similar to equation (3), the value of the call option on the average price over [T–1,T] at time 0 for the lognormal distribution can thus be expressed as,

(6) $$\begin{aligned} Z(S_0,X,r,t,\sigma) &= [(\exp(\mu)-1)/\mu]*\{S_0*\exp(\mu*(T-1)-rT)*N(d1) - \\ &\quad \exp(-rT)*X*(\mu/[\exp(\mu)-1])*N(d2)\} \\ &= \{S_0*\exp(\mu*(T-1)-rT)*N(d1)\}*[\exp(\mu)-1]/\mu - \\ &\quad \exp(-rT)*X*N(d2) \end{aligned}$$

where $$d1 = [\log(S_0*\exp(\mu(T-1))/(X*\exp(-r(T-1)*\mu/[\exp(\mu)-1]))$$

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[10]Formally, giving the law of large numbers, we know that the drift in A will be: $dA_t = S_t dt$ ($t<T$), which is not stochastic. (The drift in S itself will of course be stochastic.) There is no instantaneous uncertainty in A_t, the option will thus not need to hedged by "positions" in S, and its price will be independent of the uncertainty (standard deviation) of S.

[11]At time t ($T-1<t<T$) the expectation of the average price over the period [T-1,T] will be the average price up to t (scaled down), $A_t*(t-(T-1))$, plus the expectation of the expected average price over the remaining period, $S_t*[\exp(\mu(T-t)-1]/\mu$.

$$+ (\sigma^2/2)*(T-1)]/(\sigma\surd(T-1))$$

$$d2 = d1 - (\sigma^2/2)*(T-1)]/(\sigma\surd(T-1))$$

The closed form solution for the value of an option on the average price allows us to check the commonly made assertion that options written on average prices are lower valued than options written on end of period (maturity date) prices of the same asset because option values are an increasing function of volatility and the volatility of the average price is less than the volatility of the spot price. Comparing equation (6) with the formula for a standard European call option on the spot price with maturity T would allow us to verify this assertion. A standard European call option on the spot price at time T would be priced as:

$$(7) \qquad C(S_t,X,r,t,\sigma) = S_0*\exp((\mu-r)T)*N(d1) - \exp(-rT)X*N(d2)$$

where
$$d1 = [\log(S_0*\exp(\mu T))/X) + (\sigma^2/2)*T]/(\sigma\surd T)$$

$$d2 = d1 - (\sigma^2/2)*T]/(\sigma\surd T)$$

Comparing equation (6) and (7), and using the standard partial derivatives for the Black-Scholes option pricing formula, we find the following factors which will lead to a difference between the value of an option on the average and an option written on the end of period price. The factor $[\exp(\mu)-1]/\mu$ (multiplying the option value) is larger than one and will therefore increase the value of Z relative to C. The exercise price is lower for Z (by the factor $\mu/[\exp(\mu)-1]$) which will increase the value of Z relative to C. And the maturity for Z is shorter which will lower the value of Z relative to C. The net effect of these three factors will be unclear.[12]/

III. The Mexico 1990 Recapture Clauses and the Behavior of Oil Prices

The exact formulation of the recapture clause in the Mexico agreement was as follows. Banks choosing the debt relief options in the agreement are eligible for recovering some of the money given up through a recapture clause. Under this clause, beginning July 1996, 30% of the extra oil revenues Mexico gets if the price of oil rises above $14 per barrel (adjusted for US inflation), will accrue to the banks that have granted debt service relief. This amount is in no year to exceed 3% of the nominal value of the debt exchanged for these bonds at the time of the exchange (i.e. there is no indexation of this cap). The amount available under this clause will be scaled back by the percentage of the total debt brought under the two debt relief options.

The value at the maturity dates of the recapture clause depends therefore on the following three variables: oil export quantity, oil price behavior—which determine Mexico's oil earnings and, together with other capital flows, the

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[12]In the application for Venezuela we found that the value of the end of period price option was slightly higher than the value of the average price option.

amount of foreign exchange available to service the recapture clause—and the behavior of inflation rates—which determine the nominal exercise prices. To determine the current value of the recapture clause, we need to assess the characteristics of the stochastic process governing oil exports, oil prices and the inflation rate used for adjusting the recapture clause strike price. It is projected that the quantity of oil produced by Mexico will remain at its current level over the near future (1.2 million barrels per day, and will decrease in the late 1990s (to 0.8 million barrels a day). Little uncertainty exists regarding the projections of oil quantities exported given the long lead times from oil discovery to oil exploration and exportation, and the fact that no new oil resources are expected to be found.[13]/

The main uncertainty is introduced through the oil price. The standard deviation of the average price of Mexican oil over the last 8 years has been 23%. Similar standard deviations have been observed for prices that are close substitutes of Mexican oil, such as Borneo light (25% over 87–89), and for the average OPEC oil price (40% over 87–89, 21% over 85–89). The standard deviation of the annual changes in most (nominal) oil prices over the period 1975–1988 has been at least 20% annually. Correcting for any trend in oil prices does not change these estimates significantly and the lower bound remains 20%.

The historical estimate of the standard deviation assumes that the underlying stochastic process is expected to be the same in the future. Another way to get an estimate of the expected standard deviation is to use market information, such as actual market prices of oil options which are exercisable at a particular maturity date, to derive expected standard deviations. Given a pricing model, observed option prices can be used to back out the market expectation of the (average) volatility over the time horizon up to the exercise dates that is consistent with those prices. Doing that one finds that the historical estimates of the standard deviation of oil prices are in line with the market expectations of future volatilities as implied by the prices of options on oil traded on exchanges. Using for instance the Black and Scholes formula on recent oil option contracts results in implied volatilities for 1 to 2 year maturities of around 20%. Since historical values for the volatility of oil prices approximate closely the market's assessment of future volatility, we used the historical volatility in our pricing exercises.

The third element of uncertainty is the inflation rate used to convert the nominal oil prices into real prices as the trigger for the recapture is defined in real terms (but the cap is in nominal terms). One way to deal with the fact that the triggers are in real terms is to st[illegible] the formula for the recapture clauses in nominal terms. This implies that the exercise prices L and U [illegible]ill be

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[13]Furthermore, there does not appear to be a serious incentive problem-- the possibility that Mexico is tempted to reduce oil exports or exploration -- since the share of exports earnings going to creditors will remain small and since most of the investment to generate the earnings has already occurred. The amount of foreign exchange available is as such a non-traded asset. However, because it can easily be replicated throu[illegible] a portfolio of oil-price sensitive assets (say oil futures) and fixed-income securities which are traded, we are allowed to treat it as a traded asset.

stochastic and results in call pricing formulas which are similar to those derived by Fischer (1978) and Margrabe (1978). It is easy to show that this is equivalent to expressing the recapture clause complete in real terms and using the real oil price behavior as the relevant underlying state variable. Given that the uncertainty in the nominal oil price behavior has been significantly higher than the uncertainty in the inflation rate over the last decade, i.e. the uncertainty in the real oil price has been of the same order as the uncertainty in the nominal oil price, we considered that for practical purposes little value was added by introducing inflation uncertainty.[14]/ We used therefore simple projections of inflation rates.[15]/

There is an additional element of risk to the recapture clause. It is conceivable that the recapture clause is "in the money" when Mexico's net foreign exchange available is only sufficient to service all official sector, bonds, and (restructured) commercial banks claims, but not the recapture clause. After all, the resources available to pay out on the recapture clauses will be determined after all other creditors and (restructured) commercial debt is serviced and the recapture clause does not stipulate what happens if resources fall short. This problem can be corrected by calculating the amount necessary to service commercial bank and other claims, converting this into a minimum oil price necessary to generate that amount of foreign exchange, and imposing this as another lower bound on the recapture clause through a call option.[16]/

The value of the recapture clause per unit of debt also depends on the amount of debt converted: we use the actual amount converted under the agreement: $49 billion.

Using the above set of conditions and parameters, we can calculate the current value of the recapture clause. As our "base case" we use a standard deviation of the oil price which increases over time with the square root of time (as if the oil price where a lognormal process over time). i.e., $\sigma_t = \sigma_0 * \surd t$. The first year standard deviation was chosen as 20%. The drift in the oil price was set equal to the inflation forecast, which was 5 percent, so that the real oil price was expected to remain constant. The recapture clause was assumed to become indexed at a starting (real) oil price of $14 per barrel (the average price in

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[14]The standard deviation of the logarithm of the nominal price of Mexico's oil deflated by the U.S. price index over the last 8 years was 48%. After correcting for time trend the standard deviation was 18%.

[15]In practice, we projected the drift in the real oil price.

[16]Let N be the amount of foreign exchange for which the (new) commercial bank claims can just be serviced after all other claims are serviced. Under the assumption that the total amount that can be repaid under the clause remains capped at M, the recapture clause can be represented as: $\Sigma_{\tau'>\tau}\ \alpha*(\max[FX_{\{\tau'\}}-D,0] - \max[FX_{\{\tau'\}}-U',0])$, where $D = \max(L,N)$, which is non-stochastic, and $U' = D+M/\alpha$. Alternative, the recapture clause may only pay out up to the original maximum amount of foreign exchange, U, in which case the clause can be represented as $\Sigma_{\tau'>\tau}\ \alpha*(\max[FX_{\{\tau'\}}-D,0] - \max[FX_{\{\tau'\}}-U,0])$. and the maximum payment will be M-(D-L).

1989). The value for the recapture clause (per unit of converted debt) which resulted turned out to be 1.93 cents, or less than one-eights of the present value of the maximum contractually possible payments.[17]/

One disadvantage of the lognormal distribution is that for long time horizons this leads to very "fat" tails in the probability density of the price, something may drive the results too much.[18]/ It is analytically unclear what the effect of a high standard deviation of oil prices is on the value of the recapture clause since the value is the difference between the values of two call options, options which are both increasing functions of the standard deviation. Whether the difference increases or decreases when the standard deviation increases depends on the exact value of the parameters.

To check the sensitivity of the value of the recapture clause to the standard deviation of the oil price, we used different assumptions about the behavior of the standard deviation of the oil price over time. The base case assumed—in line with the lognormality assumption—that the standard deviation increased with the square root of time, i.e $\sigma_t = \sigma_0\surd t$. As an alternative, we first assumed that σ^2_t increases with the square root of time (i.e., $\sigma^2_t = \sigma^2_0\surd t$), implying that the standard deviation increased less fast over time as under the base case. Other parameters remained as in the base case. The value of the recapture clause *increased* in that case from 1.93 cents to 3 cents. Evidently, a decrease in uncertainty increased the value. Second, it was assumed that the standard deviation remained constant over time at σ_0. In that case, the value of the recapture clause *rose* further to 4.5 cents. Third, we used different values for σ_0. The results were similar in that the values of the recapture clause *rose* when σ_0 fell. All these sensitivity analyses indicate that the assumption of a lognormal distribution for the oil price does not lead to an overvaluation of the recapture clause. If anything, the recapture clause is undervalued if a lognormal distribution is assumed instead of a stationary distribution.

A second round of sensitivity scenarios centered on the drift of the real oil price, i.e. the drift in the oil price relative to the drift in the inflation rate. The base case assume that the real oil price remained constant, i.e. that the convenience yield was equal to the real interest rate. Assuming, instead of a flat real oil price, an annual one percent increase in the real oil price, led to the result that the value of the recapture clause increased marginally, from 1.92 cents to 2.12 cents.

A last round of sensitivity analysis centered on different values for the first year oil price.[19]/ The base case assumed a first year price of 14 dollar

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[17]As a comparison, the market value of the Mexico debt restructuring bonds in February 1990 was around 40 cents (on a old face value of 1 dollar), implying that the recapture clause represented around 5% of the value.

[18]It is arguable that oil prices are more bounded and follow for instance a mean-reverting process.

[19]The exercise price is based on Mexico's oil earnings over the last 12 months and thus depends on the average oil price over the last 12 months.

a barrel. Changing this from 14 to 16 dollar hardly changed the value of the recapture clause.

IV. The 1990 Venezuela Recapture Clause

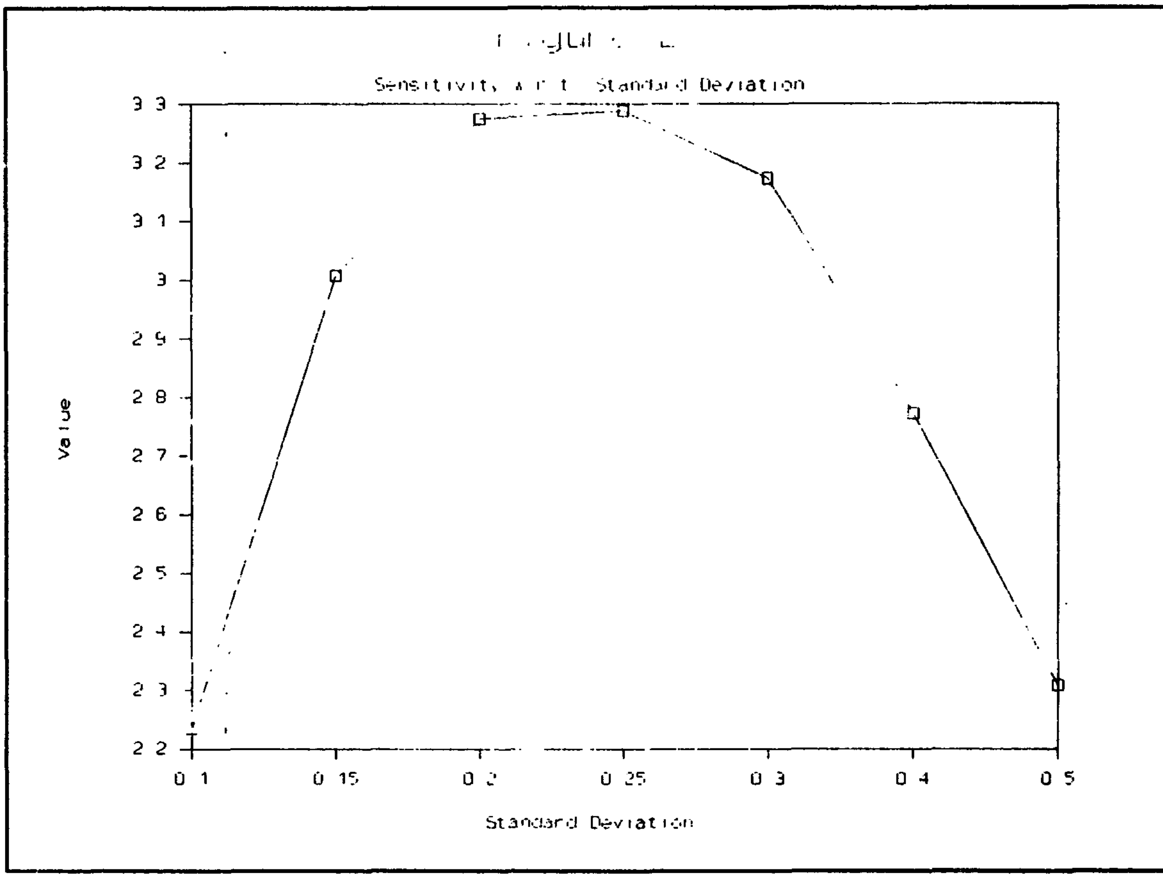
The 1990 Venezuela agreement includes a recapture clause which is similar to the one in the 1989 Mexico agreement. The exact formulation is as follows. Those banks choosing to convert their debt to defeased par bonds and defeased principal discount bonds (two of the five debt reduction options) will receive for every $1000 dollar of face value of debt exchanged five (fully detachable) warrants. Each of these warrants provides for payments by Venezuela, at each semi-annual interest payment on the bonds, beginning at the 12th and ending at the last (60th) interest payment, if the average price of oil exported by Venezuela during the preceding six months, exceeds the strike price. The strike price for the first date is set at U.S. $20.50 multiplied by $(1.02)^{12}$. The next (48) strike prices will be adjusted by the U.S. producer price index. Payments will equal the difference between the average oil price and the strike price, subject to an ceiling of $3 per warrant.

The main difference with the Mexico recapture clause is that Venezuela's export volume does not play a role for the pricing as the underlying state variable is the average price of oil and not the (average) oil exports earnings. The stochastic process of the oil price is assumed to be the same as for Mexico. The base case therefore used an expected annual standard deviation of the oil price of 20 percent and no expected annual drift in the real oil price, which, as shown, are fairly conservative parameters. Other inputs were the following. The spot price of Venezuela's exports of oil was around $16 per barrel. The interest rate used was 10 percent and the assumption for the rate of U.S. producer price inflation was set to 4 percent.

This set of inputs and assumptions leads to a base case price for one warrant of $6.6 dollars, or for 5 warrants and expressed in cents per dollar of face value of new debt, 3.3 cents. To put this in perspective, we can compare this to the ceiling payments of $3 over the period from the 12th till the last interest payment date. The present value of this series is $30 dollar per warrant or 15 cents per $1 of converted debt, implying that the recapture clause is worth less than one-quarter of the contractual maximum present value possible.

As in the case of Mexico, the value of the recovery clause is likely sensitive with respect to the following three parameters: the expected standard deviation of the oil price, the drift in the real oil price, and, of course the interest rate used. The sensitivity of the value (in cents) with respect to the standard deviation is shown in figure 2. Interesting here is that the value never exceeds 3.3 cents and that reaches this maximum of 3.3 cent for a standard deviation between 20-25 percent. The "fatter" tails of the lognormal distribution evidently do not lead to an overestimation of the price.

To check the sensitivity with respect to the other parameters some more simulation were performed. The following ranges of the other parameters led to prices that were between 2 and 4 cents per dollar of face value of new debt: interest rates between 8.5 percent and 14 percent; current oil prices between $13 and $18; producer inflation rates between 2 percent and 8 percent; and annual



drifts in the real oil price between -1.75 percent and 0.85 percent. On the basis of these sensitivities and ranges for parameters, the base price of 3.3 cents seems a fairly robust figure.

Conclusions

This paper has derived closed form solutions for the pricing of options on average prices and recapture clauses. On this basis, the values of recapture clauses in the Mexico and Venezuela agreements under alternative assumptions regarding the state variable underlying the clauses are estimated. The paper shows that the sensitivity of the values of the recapture clauses with respect to the stochastic process of the underlying variable is different than expected. The more "stationary" the process driving the underlying variable becomes, the more vaiuable the recapture clause becomes. The reason is that the effect of an increasing variance on the value of the recapture clauses is analytically unclear since in the two agreement the clauses are "collars", bounded above and below. Only in the empirical application could we find that increasing the variance reduces the value of the collar.

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